Rencontre Poisson à La Rochelle

Pol's Fest

Amphi 100 MSI

Tuesday 30.05 **Chairleader**: V. Salnikov

14:00 – 14:50: **C. Roger**
Graded Lie algebras revisited

GLA have had a long and extensive history, sometimes under the nickname of superalgebras, and important applications, either algebraic (deformation theory), geometric (e.g. Poisson geometry), or in theoretical physics (field theory and statistical physics). We shall recall the algebraic birth of some of those GLA when defined as algebras of derivations of some graded rings, and also the technology of vanishing square objects (equivalently Master Equation [Q,Q]=0 or Maurer-Cartan equation) in order to work out cohomologies and deformations.
We shall make explicit the case of Gerstenhaber algebras as developed by Murray Gerstenhaber in his founding articles in the Annals during the 60's; and give some recent computational results.

Bibliography :

- About Gerstenhaber algebra in dimension one, C. R., preprint, 2019.
- About Lie-Rinehart superalgebras, C. R., *Bulletin de la Société Royale des Sciences de Liège*, 2020, 89, pp.186-197.

CAFÉ

15:30 – 16:20: **A. Hone**
An infinite sequence of Heron triangles with two rational medians

Triangles with integer length sides and integer area are known as Heron triangles. Taking rescaling freedom into account, one can apply the same name when all sides and the area are rational numbers. A perfect triangle is a Heron triangle with all three medians being rational, and it is a longstanding conjecture that no such triangle exists. However, despite an assertion by Schubert that even two rational medians are impossible, Buchholz and Rathbun showed that there are infinitely many Heron triangles with two rational medians, an infinite subset of which are associated with rational points on an elliptic curve E(Q) with Mordell-Weil group Z x Z/2Z, and they observed a connection with a pair of Somos-5 sequences. Here we make the latter connection more precise by providing explicit formulae for the integer side lengths, the two rational medians, and the area in this infinite family of Heron triangles. The proof uses a combined approach to Somos-5 sequences and associated Quispel-Roberts-Thompson (QRT) maps in the plane, from several different viewpoints: complex analysis, real dynamics, and reduction modulo a prime.

16:30 – 17:20: **D. Dehainsala**
Separability and a Lax equation for C_2^1 *Toda lattice*,

We consider the Toda lattice associated to the twisted affine Lie algebra C_2^1. It is well known that this system is a two-dimensional algebraic completely integrable system. By using algebraic geometric methods, we give a linearisation of the system. Finally, a Lax representation in terms of 2 x 2 matrices is constructed for this system.

Wednesday 31.05 **Chairleader**: A. Pichereau

9:30 – 10:20: **V. Ovsienko**
Shadows of numbers: supergeometry with a human face,

In this elementary and accessible to everybody talk I will explain an attempt to apply supersymmetry and supergeometry to arithmetic. The following general idea looks crazy. What if every integer sequence has another integer sequence that follows it like a shadow? I will demonstrate that this is indeed the case, though perhaps not for every integer sequence, but for many of them. The main examples are those of the Markov numbers and Somos sequences. In the second part of the talk, I will discuss the notions of supersymmetric continued fractions and the modular group, and arrive at yet a more crazy idea that every rational and every irrational has its own shadow.

CAFÉ

10:50 – 11:40: **C. Evripidou**
Lotka-Volterra systems and Graphs,

There is a well-known relation between Lotka-Volterra systems and graphs. For a Lotka-Volterra system we associate the directed graph with adjacency matrix the interaction matrix of the system. It is clear that two isomorphic graphs correspond to isomorphic Lotka-Volterra systems in the sense that there is a simple smooth map between their phase space which preserves their usual Hamiltonian structure. We prove that the converse is also true, namely that two Lotka-Volterra systems are isomorphic if and only if their corresponding graphs are isomorphic. In order to prove that result we introduce the central notion of decloning of Lotka-Volterra systems and we prove that upon decloning, integrability (or non-integrability) is preserved.
This powerful result can be used for classifying the Lotka-Volterra systems having a specific property, by translating the property and working on graphs which is usually easier. We classify, using the above result, all Lotka-Volterra systems having the Kahan-Poisson property which means that their Kahan map is Poisson with respect to the original Poisson structure of the system.

11:50 – 12:40: **C.Ospel**
Generalized dendriform algebras, NS algebras and operators,

Classical constructions associate to some types of algebras (associative, Lie, preLie, Jordan,...) new algebras called dendriform, tridendiform, and NS-algebras. They can be obtained from different operator (Rota-Baxter, Nijenhuis,...).
In this talk a general construction of such algebras will be presented. It is based on joint work with P. Vanhaecke (Poitiers) and F. Panaite (Bucharest).

14:00 – 14:50: **R. Loja Fernandes**
Hamiltonian spaces of symplectic toric bundles,

I will discuss a generalization of symplectic toric manifolds, which includes, for example, the manifold $T^2 \times S^2$. For these generalized toric manifolds, the acting torus is replaced by a symplectic torus bundle, and the Delzant polytope in $\mathbb{R}^n$ is replaced by a "Delzant subspace" in an integral affine manifold. After introducing them, I will discuss a Delzant-type construction and obstructions to the existence of invariant Kähler metrics. From the point of view of integrable systems, these correspond to systems whose only singularities are elliptic. This talk is based on joint work with Maarten Mol (Max Planck, Bonn) in progress.

15:00 – 15:50: **A. Frabetti**
Direct connections on jet groupoids,

Gauge fields are the local expression of a principal connection on a principal bundle, and therefore encode the infinitesimal data of a parallel transport between the fibres along curves on the base manifold. One may wonder if there is a field interpretation for parallel transport. Reading principal connections as infinitesimal connections on the associated Atiyah Lie algebroid, this question can be answered by usual integration and gives rise to direct connections on Lie groupoids [A. Kock 1989, N. Teleman 2004, J. Kubarski and N. Teleman 2006].
In this talk we review the basic facts about direct connections on Lie groupoids, together with some interesting examples due to Teleman, and study their jet prolongations on jet groupoids.

The talk is based on a work in progress with S. Azzali, Y. Boutaib and S. Paycha.

CAFÉ

16:30 – 17:20 : **A. Vanhaecke** *The p-adic upper-half plane*,

In this talk I will explain how the p-adic analog of the complex upper half-plane works. I will stress the similarities and the differences between the two spaces. The complex upper-half plane has the particularity of being a period space and a uniformizing space which is conformally equivalent to the open unit disc. Through these three topics, I will stress the similarities and the differences between these two spaces. This will be the occasion to introduce several p-adic objects in order to give a glimpse into the p-adic realm but also to explain some recent developments concerning this space.

17:30 – 18:30: Free Poisson Discussion

20:00 Congress dinner

Thursday 1.06 **Chairleader**: V. Salnikov

10:00 – 10:50: **C. Laurent-Gengoux**
Coming from integrable systems

11:00 – 11:50: **P. van Moerbeke**
The surprising mathematics of tiling models,

In this lecture I will discuss random coverings of large geometrical domains (convex and non-convex) with dominos or lozenges. For large size domains these « tilings » will typically lead to different regions within the domain, separated by algebraic curves ; these regions can be solid, liquid or gas-like. The fluctuations of the tiles near those curves and their singularities obey universal probability laws.