Research statement

Research topics and collaborators

LMNO, Group GM3N, Université de Caen Basse-Normandie
Period: 2013 –

- Generalized geometry in gauge theories – collaboration with T.Strobl, A.Kotov.
- Improvement of thermal and electric conductivity of composites within the framework of stochastic homogenization – collaboration with P.Karamian, D.Choï (ACCEA project).

Mathematics laboratory – INSA de Rouen
Period: 2012 – 2013.

- Dynamical systems: efficient implementation of the algorithms of qualitative analysis; Numerical aspects: GPU et parallelization.
- Integrability and control theory – collaboration with W.Respondek
- Applications of the qualitative analysis of dynamical systems to problems from biology and physics – collaboration with A.Tsygvintsev.
- Computation on the graphical processors and parallelization in imaging – collaboration with C.Le Guyader.

Institit Camille Jordan Université Claude Bernard Lyon 1
Period: 2008 – 2012.

- Generalized geometry and super geometry: application to the sigma models – collaboration with T.Strobl
- Supersymmetric models – collaboration with T.Strobl, J.-P.Michel, G.Bonavolonta.

A.A.Dorodnitsyn Computing Center of the Russian Academy of Sciences
Period: 2008 – 2011.

- Dynamical systems and dynamical integrability: algebraic, geometric and topological approaches.
- Dynamical integrability: possibilities of employing the numerical methods – collaboration with S.Stepanov.

Department of pure and applied mathematics
of the École Normale Supérieure de Lyon
Period: 2006 – 2007.

- Dynamical systems and dynamical integrability: algebraic and topological approaches – collaboration with A.Tsygvintsev
- Quantization problems – collaboration with T.Strobl.

Department of Mechanics and Mathematics of the Moscow State University
Period: 2003 – 2008.

- Dynamical integrability of mechanical systems, geometric and topological approaches – collaboration with V.L.Golo.
- Modeling of the thermodynamic equilibrium for biological simulations – collaboration with V.L.Golo, K.V.Shaitan and the group of molecular simulations of the Moscow State University: `http://www.molsim.org/`

Detailed description of research interests

1 Qualitative analysis of dynamical systems

Motivations

My interest to this subject was inspired by some non-linear effects observed in the numerical simulations of the mechanical systems with constraints within the framework of the study of their thermodynamical properties. We have started by analysis of such systems even not couples to the environment, namely by analysis of their integrability.

The question of integrability of dynamical systems has been studied seriously since the middle of the XIX century. At that time the main property of the systems of ordinary differential equations that was interesting for the mathematicians was *integrability by quadratures*, i.e. the possibility to obtain a solution of a given system of ODE by inverting the functions and computation of the primitives. There are two important properties of the system that can simplify the solution of this problem: the existence of a continuous group of symmetries of the system (Noether theorem), and the existence of the functions constant along the trajectories – the first integrals (Liouville-Arnold theorem) (see e.g. *"Symmetries, Topology and Resonances in Hamiltonian Mechanics"* by V.V. Kozlov, Springer, 1996, for a review of the results).

These observations have been regrouped in the "modern" definition of integrability in the Liouville–Arnold's sens, namely the systems with n degrees of freedom should possess n independent first integrals commuting in the sens of the Poisson bracket. This formulation permits to profit from the powerful tools of the symplectic geometry and complex analysis to study the qualitative behaviour of the systems of ordinary differential equations. Integrable systems are in some sens more regular. That is for such a system it is reasonable to discuss perturbations and stability. But if the system is not integrable (and the majority of the systems are not), it means that its behaviour is qualitatively complicated and one can expect the presence of non-linear effects, chaotization, etc.

So there are two natural directions of the study of dynamical systems: the search of non-artificial integrable systems and the rigorous proof of non-integrability of the systems interesting for applications. My research in the domain concerns both problems, as well as the relations that one can establish via geometry between integrability, qualitative behaviour of dynamical systems in general and control theory in particular. An important part also addresses the efficient implementation of the methods and applications in physics, mechanics and biology.

Results and work in progress

The major part of this work has been done within the framework of my studies at the ENS de Lyon, the Moscow University and of my Ph.D. thesis in the Computing Center of the Russian Academy of Sciences. This work is being continued in the Mathematics Laboratory of the INSA de Rouen.

Topology of integrable systems.

The first natural question to be asked in the context of integrability concerns hamiltonian systems with two degrees of freedom. We have developed a rather general method to visualize the phase space to test the existence of additional first integrals for the systems that can be reduced to the phase space of small dimension ("method of sections", [7]).

We have suggested a generalization of the "method of sections" ([4]), based on the results of the Kolmogorov–Arnold–Moser theory, to analyze the real integrability of systems with parameters. This method permits in particular to localize the possible integrability regions in the parameter space.

Algebraic methods.

Since for a given dynamical systems there is no general known criteria of integrability, a natural idea is to restrict the class of "admissible" first integrals. For example one can consider the class of polynomial functions or ones admitting a meromorphic continuation; that is rather natural since for known integrable systems such a situation often takes place. This permits to employ advanced algebraic methods, namely to study algebraic groups associated to variational equations along a particular solution of the initial system. A rather detailed review can be found for example in *"Hamiltonian systems and their integrability"* by M. Audin (Cours Spécialisés, SMF et EDP Sciences, 2001). The key idea is that for an integrable systems some groups (the monodromy group in the approach of S.L.Ziglin and the differential Galois group in the approach of J.Morales and J.-P.Ramis) cannot be too complicated, and even often should be commutative.

The approaches of Ziglin and Morales–Ramis need an explicit particular solution of the dynamical system. Since for a given solution the differential Galois group contains the monodromy group, it is considered that the Morales–Ramis method is stronger than the Ziglin's one. We have considered the possibility to use a trajectory obtained numerically to construct the monodromy group and apply the latter method which is qualitatively more comprehensible. This permits to formulate an effective algorithm to search for the obstructions to meromorphic integrability ([3]). It is rather natural to suggest the usage of numerical methods of this type to study the properties of dynamical systems, but there are surprisingly few works on the subject, all addressing rather the computation of the groups along a known solution. Our approach permits to overcome also the difficulty of construction of this particular solution, allowing in addition multivalued solutions.

Applications and efficient implementation.

We have applied these two methods to analyze the integrability of systems having mechanical origin. Namely we have shown that the pendulum-type systems are integrable only in the trivial cases when the systems decouples to parts of smaller dimension. We have also shown the non-integrability of the double pendulum with gravity, of the axisymmetric satellite, and the Henon-Heiles system ([2 – 4, 6, 7]).

There are also some activities in progress related to this research topic. In particular they concern the qualitative analysis of the systems coming from "relativistic" celestial mechanics and modelling in biology. The first one is the consideration of a natural modification of the problem of gravitating bodies with a finite speed of propagation of interaction, that results in a dynamical system with delay. And the first natural question is to classify the possible bounded solutions. In the classical case they correspond to the trajectories more regular than the generic ones. In what concerns the problem from biology, this is the analysis of the models of population dynamics adapted recently also to the simplified description of the immune reaction in the context of cancer-type diseases. For these systems an important problem is to search for the stationary points or the solutions of the limit-cycle type. For these examples the mentioned effects can be revealed be the methods described above.

Since the discussed methods are partially based on the numerical computations one should not neglect the implementation details. We have applied the CUDA technology of parallel programming for the GPU, that seems to be an optimal solution on this case ([1]).

Other directions.

The work in progress concerns mainly the links between the integrability and other domains of qualitative analysis of dynamical and control systems. We study the generalization of the approach of [3] for the differential Galois group as well as for the q-difference equations. We also explore the relation between the general theory of integration of algebroids with the approach of B. Malgrange to dynamical integrability For the control theory we study the flat systems and the link between flatness and integrability in the Frobenius sens, namely we would like to formulate a constructive method of verifying and eventually visualizing the flatness propoerty.

Diffusion

1. V.Salnikov, *Intégrabilité dynamique: de l'approche algébrique au calcul parallèle*, Matapli (SMAI), N° 100, 2013 (in French).

2. V.Salnikov, *Integrability of the double pendulum - the Ramis' question*, arxiv:1303.4904 [math.DS].

3. V.Salnikov, *Effective algorithm of analysis of integrability via the Ziglin's method*, Journal of Dynamical and Control Systems, March 2014 DOI: 10.1007/s10883-014-9213-z.

4. V.Salnikov, *On numerical approaches to the analysis of topology of the phase space for dynamical integrability*, Chaos, Solitons & Fractals, Vol. 57, Dec. 2013.

5. V.Salnikov, Ph.D. Thesis defended in the Computing Center of the Russian Academy of Sciences, 2011.

6. Vl.N.Salnikov, *On the dynamics of a triple pendulum: Various approaches to non-integrability*, Proceedings of the Contest-Conference for Young Scientists, Institute of Mechanics of MSU, 4 pages, 2007, (in Russian).

7. V.Salnikov, *On the dynamics of the triple pendulum: non-integrability, topological properties of the phase space*, Lecture notes of The Conference "Dynamical Integrability", 11 pages, 2006.

2 Generalized geometry in physical theories

Motivation

My research activity within the framework of this subject consists of a rather theoretical study related to the tools of modern differential geometry in the context of gauge theories.

The study of gauge theories is a subject rapidly developing since the middle of the twentieth century. On the one hand this is a powerful tool to explain the dynamics of elementary particles, permitting also to define quantization procedures; on the other hand this topic is often related to rich geometric structures and even motivates their definitions as it was the case for super geometry.

Independently of the applications to physical theories, graded geometry permits to give a unified description of several objects from modern geometry, such as vector bundles, symplectic and contact manifolds, algebroids, etc.

Results and work in progress

This subject interests me since my stay at the ENS de Lyon as a master second year student. This is also the main contents of my Ph.D. thesis defended in the UCB Lyon 1

Graded and multigraded geometry

A Q-manifold is a graded manifold equipped with a homological vector field of degree 1, that we will call a Q-structure. One can associate natural Q-structures for example to a tangent bundle, a Poisson manifold, or more generally to a Lie algebroid. For the Q-manifolds we have defined a notion of equivariant Q-cohomology generalizing the description of ordinary equivariant cohomology via the Weil or the Cartan algebra.

We have also performed some constructions for multigraded manifolds, namely Q-structures, graded symplectic structures and measure compatible with this objects. This permits to formulate a possible generalization to multigraded case of the result of Aleksandrov–Kontsevich–Schwarz–Zaboronsky that states the existence of a symplectic structure on the space of maps between two graded manifolds equipped with some compatible geometric structure.

Sigma models.

In a very general sens, a sigma model is a functional on the space of maps between two manifolds equipped with some geometric structures. In the physical interpretation the starting manifold plays the role of the space-time and the arrival manifold (target) carries the information on the physical field content.

We have adapted a convenient approach of A.Kotov and T.Strobl to "encode" the essential ingredients of the sigma models employing the notions coming from the Q-manifolds and Q-bundles. This permits to obtain a geometric description of the symetries of the twisted Poisson sigma model (PSM, [3]) and the Dirac sigma model (DSM, [2]).

Inspired by the relation of gauging the Wess-Zumino term in the G/G WZW model with equivariant cohomology we implemented a likewise idea to the twisted PSM and the DSM ([2, 3]). Using the notion of equivariant Q-cohomology we have shown that (topological parts of) these models can be recovered from an equivariantly closed extension of the 3-form of the Wess-Zumino term. An important role in this context is played by an extension of the infinite dimensional Lie algebra corresponding to the Dirac structure. This Lie algebra describes the gauge transformations of the obtained models. The fact that the construction is rather similar for the twisted PSM and the DSM is related to the geometry of the Lie and Courant algebroids, as well as the Q-structures appearing naturally in the context. Moreover, the DSM is the most general sigma model in space-time dimension of 2 that can be obtained by gauging the symmetry group of the Wess-Zumino term ([5]).

Supersymmetric theories

For the supersymmetric sigma models the space-time or the target manifold can be naturally graded (before the introduction of the Q-formalism). The appropriate language for studying such theories is then the multigraded geometry. We have analyzed some supersymmetric gauge theories ([4]). After having properly defined all the objects from mathematical point of view, we showed for example that the world-sheet supersymmetric Poisson Sigma Model is on-shell equivalent to an ordinary PSM; and for the Chern-Simons theory one can find equivalent theories with supersymmetric world-sheet or target. We have also studied some supersymmetrization techniques and their relation to the generalization of the result of Aleksandrov–Kontsevich–Schwarz–Zaboronsky.

Other directions

The suggested idea of studying the cohomology of graded manifolds and Q-manifolds is rather general. There are natural applications in theoretical physics, for example it permits to construct gauge theories with a given symmetry group or study the obstructions to gauging ([5]).

It is also useful for purely mathematical problems. There exists for example a non-trivial relation with the cohomology of Lie algebroids. It should also permit to recover within the framework of a general approach some characteristic classes obtained by physicists. Within the framework of equivariant Q-cohomology we have defined the cohomology of Courant algebroids ([3]).

Diffusion

1. V.Salnikov, Ph.D. thesis defended in the Institut Camille Jordan Université Claude Bernard Lyon 1, 2012.
2. V.Salnikov, T.Strobl, *Dirac Sigma Models from Gauging*, Journal of High Energy Physics, 11/2013; 2013(11). DOI:10.1007/JHEP11(2013)110.
3. V.Salnikov, *Graded geometry in gauge theories and beyond*, to appear in Journal of Geometry and Physics 2014.
4. *Supersymmetric AKSZ sigma models*, in preparation – collaboration with J.-P.Michel, G.Bonavolonta and T.Strobl.
5. A.Kotov, V.Salnikov, T.Strobl, *2d Gauge Theories and Generalized Geometry*, to appear in Journal of High Energy Physics, 2014.

3 Modeling

Motivation.

With this project I started my research activities as a first-year student in the group uniting the Department of Mechanics and Mathematics and the Department of Biology of Moscow State Lomonosov University. One of the main goals was to develop an efficient tool for performing biological simulations (an extremely popular subject nowadays – cf. the Nobel Prize 2013 in chemistry). A part of this group was particularly interested in modeling of the thermodynamical equilibrium for molecular dynamics. Recently I have continued this activity in the Nicolas Oresme Mathematics Laboratory of the University of Caen.

Results and work in progress.

Molecular dynamics

Combining the analytical methods coming from the dynamical systems and numerical simulations we have characterized the range of applicability of so-called mechanical thermostats ([5]). Also some particular (sometimes non-physical) regimes of dynamics of systems confined to the Nosé-Hoover environment have been found ([4,6]), that is especially important for applications.

The difficulty of analysis within the framework of molecular dynamics is due to the size of the studied systems. The main challenge was to find the systems that are rather simple for theoretical analysis but sufficiently non-degenerate to observe the qualitative effects.

We managed to formulate the criteria of applicability of the mechanical thermostats and describe the effects that are observed in the 'incorrect' application. That was extremely useful for the further work of the group (`http://www.molsim.org/`) as well as for the molecular dynamics community in general.

Effective properties of composites

This work is being carried out within the project ACCEA (Amélioration des Conductivités des Composites pour Equipements Aéronautiques), selected by FUI 15 (Fonds Unique Interministériel). This applied project has a very precise goal to develop a composite material to fulfill the demands from aeronautical industry. Among the partners in this work we have several companies that are able to produce composite materials and provide microscopic and macroscopic measurements of sample. Our team is responsible for validation and eventual optimization of suggested solutions.

Even if we can not tell much more on the applied side of the project (because of the industrial confidentiality), we have obtained some purely scientific results on the methodology of analysis of effective properties of multiphase media, this gave rise to several publications. The main method that we have adopted is based on stochastic homogenization: the idea comes from the classical Monte Carlo Method. We study the samples with the same macroscopic parameters but with some variation of microstructural morphology – for each sample we compute effective properties and the finale result is the averaged value along the whole series. In this process there are two important stages: the generation of samples of a composite material (or data collection in the applications) and the computations of homogenized coefficients. For the first problem we have developed and implemented a generation protocol, inspired in particular by the work [5]. In [1] we describe efficient procedures to generate a sample containing a mixture of inclusions of various geometries; on top of that we are able ([2]) to introduce imperfections and defects to inclusions in order to observe realistic physical effects. For the homogenization part we have adapted (and implemented) an FFT-based approach (cf. recent works of J.C. Michel, H. Moulinec, P. Suquet; V. Monchiet, G. Bonnet; D.G. Eyre, G.W. Milton). This allowed us to study the influence of morphology of inclusions on the effective mechanical ([3]) and thermal ([7]) properties. In addition we have collected a large data base that will be useful for applications, namely for estimation of parameters and resolution of various inverse problems.

Other directions

In what concerns the effective electromagnetic properties, another approach based on the analysis of percolating networks is in the process of development. We are also interested in possibilities of coupling the models in order to obtain an efficient and reliable description of thermomechanical and thermoelectric properties of composites, an even the transition effects.

Diffusion.

1. V.Salnikov, D.Choi, P.Karamian-Surville, On efficient and reliable stochastic generation of RVEs for analysis of composites within the framework of homogenization to appear in Computational Mechanics, 2015. Preprint: arXiv:1408.6074.
2. V.Salnikov, D.Choi, P.Karamian, S.Lemaitre, Génération de VER 3D par la dynamique mol eculaire et variations autour de la pixellisation. Calcul des propriétés effectives des composites, submitted.
3. V.Salnikov, S.Lemaitre, D.Choi, P.Karamian-Surville, Measure of combined effects of morphological parameters of inclusions within composite materials via stochastic homogenization to determine effective mechanical properties, submitted, Preprint: arXiv:1411.4037
4. V.N.Salnikov, *Nonlinear Dynamics in the Nosé-Hoover Environment*, Proceedings of the Fifth EUROMECH Nonlinear Dynamics Conference, 12 pages, 2005, publ. on the CD.
5. V.L.Golo, Vl.N.Salnikov, and K.V.Shaitan, *Harmonic Oscillators in the Nosé-Hoover Environment*, Physical Review, E70, 046130, 7 pages, 2004.
6. Vl.N.Salnikov, *Nonlinear Dynamics and Resonance Effects of Systems in the Nosé-Hoover Thermostat*, Proceedings of the Contest-Conference for Young Scientists, Institute of Mechanics of MSU, 4 pages, 2004 (en russe).
7. Measure of combined effects of morphological parameters and the random distribution of inclusions of type spheres-cylinders within composite materials via stochastic homogenization to determine effective thermal and electric properties, (en collaboration avec D.Choi, P.Karamian, S.Lemaitre)