INSA de Rouen

STPI - SIB - M2

Tutorial n$^{\text{o}}$ 6
Polynomials.

Exercise 1 *Horner's scheme*

1. *Perform partial division of* $P(x) = x^4 - 3x^3 + 6x^2 - 10x + 16$ *by* $(x-4)$*, and compute* $P(4)$*.*

2. *Compute* $Q(x) = x^5 + (1+2i)x^4 - (1+3i)x^2 + 7$ *at* $x = -2 - i$*.*

3. *Find the multiplicity of the root* x_0 *of the following polynomials:*

a). $x^5 - 5x^4 + 7x^3 - 2x^2 + 4x - 8$, $x_0 = 2$
b). $x^5 + 7x^4 + 16x^3 + 8x^2 - 16x - 16$, $x_0 = -2$

Decompose them into a product of irreducible polynomials over $\mathbb{R}$ *and* $\mathbb{C}$ *and sketch their graphs.*

Exercise 2 *Viète's formulas*
Compute the product of all the roots and the sum of squares of all the roots of the following polynomials:
a). $3x^5 - x^3 + x + 2$ *b).* $x^n + ax^{n-1} + b, \quad n \geq 3$

Exercise 3 *Euclidean division*

1. *Perform the Euclidean division of the following polynomials:*
a). $x^4 + x^3 - 3x^2 - 4x - 1 + 2$ *by* $x^3 + x^2 - x - 1$ *b).* x^n *by* $x - 1$

2. *Compute* $R = gcd(P, Q)$ *for* $P(x) = x^4 + 2x^3 - x^2 - 4x - 2$, $Q(x) = x^4 + x^3 - x^2 - 2x - 2$*.*
Find the linear expression of $R: R = PS + QT$*.*

3. *Can the polynomial* $R(x) = x^4$ *be a linear expression* $R = PS + QT$ *with*
$P(x) = x^4 - 2x^3 - 4x^2 + 6x + 1$, $Q(x) = x^3 - 5x - 3$*? If yes, find* S *and* T*.*

4. *Find the multiple roots of the following polynomials*

a). $x^5 - 10x^3 - 20x^2 - 15x - 4$
b). $x^7 - 3x^6 + 5x^5 - 7x^4 + 7x^3 - 5x^2 + 3x - 1$.

Find the values of the parameter a*, when the polynomial* $x^3 - 8x^2 + (13-a)x - (6+2a)$ *admits multiple roots.*

Exercise 4 *Simple fractions*
Perform the partial fraction decomposition over $\mathbb{C}$ *then where possible over* $\mathbb{R}$*.*

a). $\dfrac{x^4}{4 - 9x^2}$ *b).* $\dfrac{x^4}{4 + 9x^2}$ *c).* $\dfrac{x^3}{x^2 + 2x + 2}$

d). $\dfrac{x^5}{(x-1)(x^2-1)}$ *e).* $\dfrac{x^2(4x^3 + x^2 + 4x - 1)}{(x^2-1)(x^2+1)^2}$

Exercise 5 *Back to linear algebra (optional)*

1. *Give some examples of the subsets of polynomials that form a finite dimensional vector space with respect to the usual operations.*

2. *Consider the set* P_3 *of polynomials of degree at most* 3*. Prove that it is a vector space. What is the dimension of* P_3*? Construct a basis of it. Consider the mapping* $\frac{d}{dx}: P_3 \to P_3$, $P(x) \mapsto P'(x)$*. Using it's linearity give the matrix of this mapping in the constructed basis.*
Find $Ker(\frac{d}{dx})$*. Show that on* P_3 $(\frac{d}{dx})^{\circ 4} \equiv 0$*.*