INSA de Rouen

STPI - SIB - M2

Tutorial n° 1
Complex numbers

Complex numbers and equations

Exercise 1

Simplify the following complex numbers, cast them into algebraic form $z = x + iy$.

$$a).\ (1+3i)\overline{(7-i)} \qquad b).\ \frac{9+2i}{3-2i} \qquad c).\ \frac{1+i\sqrt{3}}{\sqrt{3}+i} \qquad d).\ \frac{\sqrt{7}+5i}{2\sqrt{7}-2i} + \frac{2\sqrt{7}-2i}{\sqrt{7}+5i} \qquad e).\ e^{i\frac{\pi}{4}}e^{i\frac{\pi}{3}}$$

Exercise 2

Cast the following complex numbers into trigonometric form
$z = r(\cos\varphi + i\sin\varphi), \quad r \geq 0, \varphi \in [0, 2\pi[$.

$$a).\ 1+i \qquad b).\ \sqrt{3}+i \qquad c).\ \frac{\sqrt{3}+i}{1+i} \qquad d).\ \frac{1+i\sqrt{3}}{\sqrt{3}+i} \qquad e).\ \frac{5+11i\sqrt{3}}{7-4i\sqrt{3}}$$

Exercise 3

Compute the following complex numbers depending on $n \in \mathbb{N}$ *and cast them into algebraic form.*

$$a).\ \left(\frac{1+i\sqrt{3}}{\sqrt{3}+i}\right)^n \qquad b).\ \left(\frac{5+11i\sqrt{3}}{7-4i\sqrt{3}}\right)^n$$

Exercise 4

Solve the following equations in $\mathbb{C}$.
a). $z^6 = 1$ *b).* $z^5 - z = 0$ *c).* $(1+i\sqrt{3})z^4 - 3 = 0$ *d).** $(z+i)^n = (z-i)^n$, $n \geq 2$

Exercise 5

The goal of this exercise is to solve in $\mathbb{C}$ *the equation*

$$z^3 - (6-i)z^2 + (13-i)z - 10 - 2i = 0 \qquad (*)$$

1. *Knowing that it exists, find the real solution* z_0 *of the equation* $(*)$.

2. *Find* $a, b \in \mathbb{C}$ *such that for all* $z \in \mathbb{C}$ *the following identity holds:*

$$z^3 - (6-i)z^2 + (13-i)z - 10 - 2i = (z - z_0)(z^2 + az + b).$$

(Later on the lecture we will formulate a more general decomposition result).

3. *Solve completely the equation* $(*)$ *in* $\mathbb{C}$.

Complex numbers and trigonometric functions

Exercise 6

For $\alpha \in \mathbb{R}$ *express* $\sin 3\alpha, \cos 3\alpha, \sin 4\alpha, \cos 4\alpha$ *as functions of* $\sin\alpha$ *and* $\cos\alpha$.

Exercise 7

1. *For* $x \in \mathbb{R}$ *express* $\cos 5x$ *as a functions of* $\cos x$.

2. *Deduce that* $\cos(\frac{\pi}{10})$ *is a solution of* $16x^4 - 20x^2 + 5 = 0$

3. *Compute* $\cos^2\frac{\pi}{10}$

4. *Deduce the values of* $\cos\frac{\pi}{5}$ *and* $\sin\frac{\pi}{5}$