INSA de Rouen

STPI - SIB - M2

Tutorial n$^{\text{o}}$ 0
Trigonometric functions – reminder

Exercise 0
Recall the notion of the trigonometric (unit) circle.

1. *Write explicitly the algebraic relations between the points on the circle that*
– coincide,
– are symmetric with respect to the coordinate axes,
– are symmetric with respect to the origin,
– are symmetric with respect to $y = x$ or $y = -x$ lines.

2. *Define the basic trigonometric functions (*$\sin, \cos, \tan, \cot$*) on the circle.*

3. *Explain the meaning of the main trigonometric identity* $\sin^2\alpha + \cos^2\alpha = 1$.

Exercise 1 *Identities.*

1. *Prove (via geometric arguments) the identity* $\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$.

2. *Write explicitly the relations (parity, periodicity, symmetries...) between the basic trigonometric functions coming from the situations of the previous exercise point **1.***

3. *From the points **1.** and **2.** deduce the formulas for*
$\cos(\alpha + \beta)$, $\sin(\alpha \pm \beta)$ *(*these you should remember*);*
$\tan(\alpha \pm \beta)$, $\cot(\alpha \pm \beta)$ *(*these you should be able to deduce fast*).*

4. *Specify the results of **1.** and **3.** to the case $\alpha = \beta$. Deduce the formulas for* $\sin^2(\alpha)$ *and* $\cos^2(\alpha)$ *in terms of functions of* 2α *(*these are useful to remember as well*).*

5. *Deduce the expressions of* $\sin(\alpha)\sin(\beta)$, $\sin(\alpha)\cos(\beta)$, $\cos(\alpha)\cos(\beta)$ *in terms of functions of* $\alpha \pm \beta$ *(*these are not necessary to remember, but should be deduced when asked*).*

6. *Deduce the splitting formulas for* $\sin(\alpha) \pm \sin(\beta)$ *and* $\cos(\alpha) \pm \cos(\beta)$ *in terms of products of functions of* $\frac{\alpha \pm \beta}{2}$. *(*There is a nice way to recover these formulas.*)*

Exercise 2 *Applications.*

1. *Compute (from geometric arguments or using the identities from previous exercises)* $\sin, \cos, \tan, \cot$ *of* $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ *(*These are to be remembered absolutely, but you probably already do*).*

2. *Solve the following equations. Depict their solutions on the unit circle.*
a). $\cos^2 x = \frac{1}{2}$, *b).* $5\sin^2 x = 2 - \cos^2 x$
c). $\sin x = \cos x$ *d).* $\tan x = \cot x$ *e).* $\sin x = \tan x$

3. *Solve the following equations.*
a). $\cot^2 \frac{\pi x}{4} = 1$ *b).* $2\cos\frac{x}{2}\sin 3x = \cos\frac{x}{2}$ *c).* $\sin 2x - \sqrt{3}\cos x = 0$

4. *Solve the following equations. Make sure that you consider equivalent equations while solving.*
a). $\frac{\sin 2x}{1 + \sin x} = -2\cos x$ *b).* $-5\cos 4x = 2\cos^2 x + 1$ *c).* $2(\cos x - 1)\sin 2x = 3\sin x$
d). $\sin^8 x - \cos^8 x = \frac{1}{2}\cos^2 2x - \frac{1}{2}\cos 2x$ *e).* $5\sin^2 x - 4\sin x\cos x - \cos^2 x = 4$
f). $\tan 3x = \tan 5x$ *g).* $\sqrt{\sin x} = \sqrt{1 - 2\sin^2 x}$

5. *Find all couples of parameters (a, b), such that the following holds for all x:*
$a(\cos x - 1) + b^2 = \cos(ax + b^2) - 1$